Problem Set 2 (*Instructor: T. Pulliam*)

1. Using Taylor Tables find er_t for
   (a) $(\delta_x u)_j = (u_{j+1} - u_{j-1})/(2\Delta x)$
   (b) $(\delta_x u)_j = (-u_{j+2} + 8u_{j+1} - 8u_{j-1} + u_{j-2})/(12\Delta x)$
   (c) $\frac{1}{6}((\delta_x u)_{j+1} + 4(\delta_x u)_j + (\delta_x u)_{j-1}) = (u_{j+1} - u_{j-1})/(2\Delta x)$

2. A second-order backward (or "upwind") differencing approximation to a 1st derivative is given as a point operator by

$$\left(\frac{\partial u}{\partial x}\right)_j = \frac{1}{2\Delta x}(u_{j-2} - 4u_{j-1} + 3u_j) \tag{1}$$

   and can be analyzed as in Chapter 3 in the class notes.

   (a) Express Eq. 1 in banded matrix form, then derive the symmetric and skew symmetric matrices that have it as their sum. Do this symbolically, using the banded matrix notation, e.g. B(m:a,b,c,d,e), for each of the terms. *(See Appendix A for constructings symmetric and skew symmetric matrices)*
   (b) Construct a Taylor table for both the symmetric and skew symmetric matrices in Prob. 2a. Find the derivative approximated by each term and er_t for both. *Note these are just two difference formulas which may or may not be approximations to derivatives. But, from the Taylor Tables you should be able to identify the type of term being approximated and the error terms*

3. Find, by means of a Taylor table, the values of a, b, c, and d that minimize the value of er_t in the expression

$$a\left(\frac{\partial u}{\partial x}\right)_{j-1} + \left(\frac{\partial u}{\partial x}\right)_j - \frac{1}{\Delta x}[bu_{j+1} + cu_j + du_{j-1}] = ? \tag{2}$$

   What is the resulting finite difference scheme and what is the value of er_t?